Topic of this homework: Prime numbers, greatest common divisors, pythagorean triples
Deliverable: Answers to questions.

1 Prime numbers

1. According to the fundamental theorem of arithmetic, every integer may be written as a product of primes.
   (a) Put the numbers 1,000,000, 1,000,004 and 999,999 in the form $N = \prod_k \pi_k^{m_k}$ (you may use Matlab to find the prime factors).
   (b) Give a generalized formula for the natural logarithm of a number N in terms of its primes π_k.
2. Prime numbers may be identified using 'sieves'
   (a) By hand, perform the sieve Eratosthenes for $n = 1 \ldots 49$. Circle each prime p then draw a slash through each number which is a multiple of p.
   (b) In part (a), which is the highest number you need to consider before all primes have been identified?
   (c) Generalize: for $n = 1 \ldots N$, which is the highest number you need to consider before all primes have been identified?

2 Greatest common divisors

Consider Euclid's algorithm to find the greatest common divisor (GCD; the largest common prime factor) of two numbers

1. Understand Euclid's algorithm
   (a) Use the Matlab command `factor` to find the prime factors of $a = 85$ and $b = 15$. What is the greatest common prime factor of these two numbers?
   (b) By hand, perform Euclid's algorithm for $a = 85$ and $b = 15$.
   (c) By hand, perform Euclid's algorithm for $a = 75$ and $b = 25$. Is the result a prime number?
   (d) Describe in your own words how the GCD algorithm works. Try the algorithm using numbers which have already been separated into factors (e.g. $a = 5 \cdot 3$ and $b = 7 \cdot 3$).
2. Write a matlab function, `function x = my_gcd(a,b)` which uses Euclid's algorithm to find the GCD of any two inputs `a` and `b`. Test your function on the (a,b) combinations from parts (a) and (b). Include a printout (or handwrite) your algorithm to turn in.

   Hints and advice:
   - Don't give your variables the same names as Matlab functions! Here, `gcd` is an existing function, so if you use it as a variable or function name, you won't be able to use `gcd` to check your own function. Try `clear all` if you accidentally do this.
   - Try using a 'while' loop for this exercise (see Matlab documentation for help).
   - You may need to make some temporary variables for `a` and `b` in order to perform the algorithm.

3 Pythagorean triples

Euclid's formula for the Pythagorean triples gives $a = p^2 - q^2$, $b = 2pq$, and $c = p^2 + q^2$.

1. What condition(s) must hold for p and q such that a, b, and c are always positive and nonzero?
2. Solve for p and q in terms of a, b and c. Hint: you don't need to use b.
3. Consider Figure 1.3 of Stillwell. Find p and q for the first five (a,c) pairs in Plimpton 322.
4. Set $n = p - q$, and find a relationship between $\sqrt{b+c}$, a, and n (you may wish to start by finding new equations for the pythagorean triples involving q and n). Is $b+c$ always a perfect square? What condition on n and a is necessary for $b+c$ to be a perfect square?